

A Hot Method for Synthesising Cool Controllers

I. Husien, N. Berthier and S. Schewe

University of Liverpool

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Motivation: Solving Symbolic DCS Problems by using General Search Techniques

Symbolic Discrete Controller Synthesis (DCS) Problem

- Given:
- System Model S with means of control
 - Desired Objective Behaviours ω
- Compute:
- Control Strategy *s.t* S controlled accordingly fulfils ω

Motivation: Solving Symbolic DCS Problems by using General Search Techniques

Symbolic Discrete Controller Synthesis (DCS) Problem

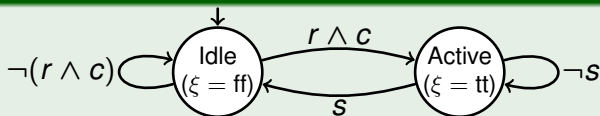
- Given:
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General Search Techniques

- Explore the space of all Candidate Solutions by performing Local Changes to candidates selected by means of a measure of their Fitness
e.g., Genetic Programming and Simulated Annealing

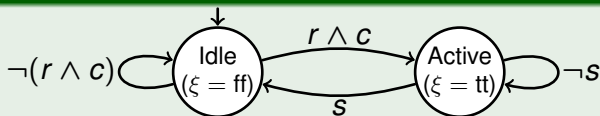
- Genetic Programming already investigated for [Program Synthesis](#) [Clark and Jacob, 2001, Henderson et al., 2003, Johnson, 2007, Katz and Peled, 2008, 2009, 2011]
 - Simulated Annealing performs better than Genetic Programming for Program Synthesis [[Husien and Schewe, 2016](#)]
- ↪ What about for solving Symbolic DCS Problems?

Two-state Task



- Non-controllable input variables: $\{r, s\}$
- State variables: $\{\xi\}$
- Controllable input variables: $\{c\}$

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Safety Objective as CTL Formula

- Never enter Active while s holds: $AG(s \Rightarrow AX\neg\xi)$

Symbolic DCS Problem

Given:

- **Symbolic Finite-state System** S with **Controllable** input variables
- **Set of Objective CTL Formulae** ω involving the variables of S

Compute:

- A **Strategy** for assigning values to the Controllable input variables *s.t* S satisfies every objective in ω

Symbolic Discrete Controller Synthesis Problem

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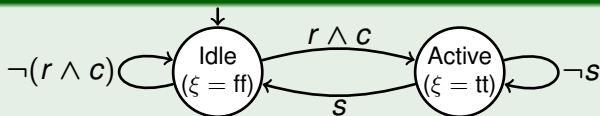
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Two Families of Symbolic Strategies

Non-deterministic Propositional Predicate involving variables of S

Deterministic Assignments to every Controllable input variable of S

Two-state Task

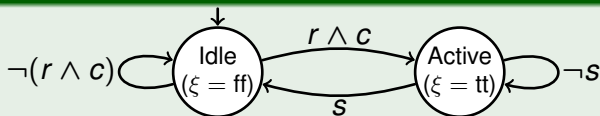


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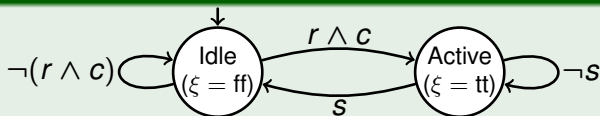
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Correct Strategies for Choosing Values of c

- Choose c s.t. $(\neg\xi \wedge s \wedge r \Rightarrow \neg c)$ holds (Non-deterministic)

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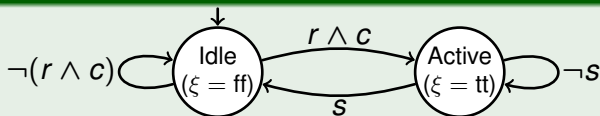
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- Choose c s.t. $(\neg\xi \wedge s \wedge r \Rightarrow \neg c)$ holds (Non-deterministic)
- Set c to $(\xi \vee \neg r \vee \neg s)$ (Deterministic)

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- Set c to ff (Deterministic)

Genetic Programming

- Maintain a Population P of λ Candidates
- Until a candidate solution is found in P :
Select $\mu \ll \lambda$ candidates based on their Fitness;
Generate new candidates by means of Mutations (and Crossovers)

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Simulated Annealing

- Temperature θ decreasing over time (“Cooling schedule”)
- Unless the Candidate x is a solution:
Mutate x to obtain x' ;
Repeat with x' instead of x if $\text{Fitness}(x') \geq \text{Fitness}(x)$,
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Issues

- How to Represent Candidate Strategies?
- How to Mutate Candidates?
- How to Measure the Fitness of a Candidate?
- How to Explore the Search Space Efficiently?

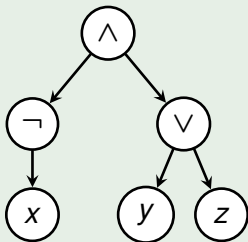
Representing Candidates

Trees built according to the Grammar of Propositional Predicates

Leaves Labelled with either a variable, or a constant in (tt, ff);

Nodes Labelled with \neg , \vee , or \wedge

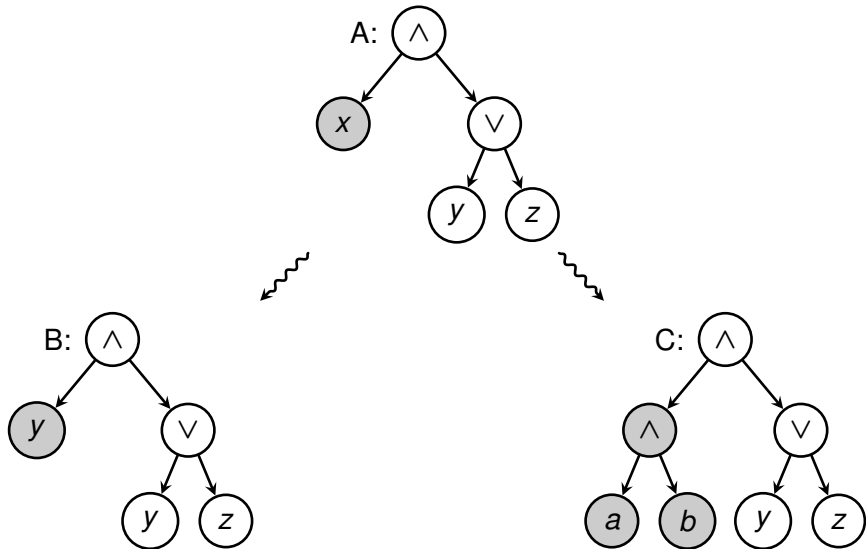
e.g., $\neg x \wedge (y \vee z)$



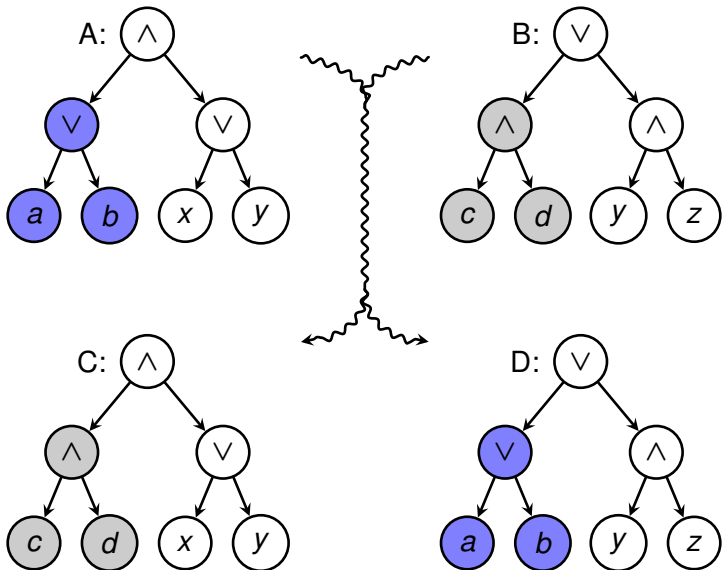
Contrary to Classical Symbolic DCS Algorithms, we directly seek **Deterministic Strategies** (i.e., Assignments to controllable variables)

↔ One candidate strategy \equiv One tree per controllable variable

How to Make Changes? Example Mutations



How to Make Changes? Example Crossover



Deriving **Partial Objectives** from Target Objectives ω

ω' Obtained by Substituting one Universal Path Quantifier occurring Positively of Objectives of ω , if any, by an Existential one:

$$A\psi \rightsquigarrow E\psi$$

ω'' Ditto with Objectives of ω'

Model Checking as a Fitness Function

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Example Derived Partial Objective Formulae

Given $\omega = \{AG(s \Rightarrow AX\neg\xi)\}$:

$$\omega' = \{EG(s \Rightarrow AX\neg\xi), AG(s \Rightarrow EX\neg\xi)\}$$

$$\omega'' = \{EG(s \Rightarrow EX\neg\xi)\}$$

Measuring the Fitness of a Candidate Strategy Γ

- Build System S' controlled according to Γ
- **Model-check** S' against objectives of ω , ω' , and ω''
- Add m points per satisfied objective of ω
- Add m' ($< m$) points per satisfied objective of ω'
- Add m'' ($< m'$) points per satisfied objective of ω''

Simulated Annealing

Rigid SA Considers all objective properties to determine fitness

Safety-first SA Safety properties are considered first

Six Search Algorithms for Solving DCS Problems

Simulated Annealing

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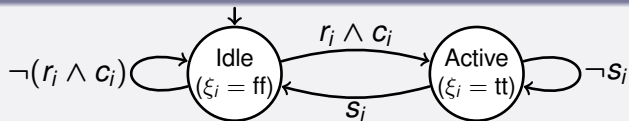
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Genetic Programming

Crossovers?

		no	yes
Safety-first?	no	GP	GP w. CO
	yes	Safety-first GP	Safety-first GPw. CO

N -Tasks



- Non-controllable input variables: $\{r_1, \dots, r_N, s_1, \dots, s_N\}$
- Controllable input variables: $\{c_1, \dots, c_N\}$

N-Tasks



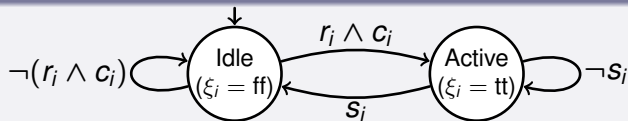
- Non-controllable input variables: $\{r_1, \dots, r_N, s_1, \dots, s_N\}$
- Controllable input variables: $\{c_1, \dots, c_N\}$

Safety Objective: *Mutual Exclusion* between Active tasks

$$\left\{ \text{AG} \left(\bigwedge_{(i,j) \in \{1, \dots, N\}^2, i \neq j} \neg (\xi_i \wedge \xi_j) \right) \right\}$$

Scalable Benchmarks

N-Tasks



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Liveness Objectives: Every Idle task *Eventually* becomes Active

$$\bigcup_{i \in \{1, \dots, N\}} \left\{ \text{AG} \left((\neg \xi_i \wedge r_i) \Rightarrow \text{AF} \xi_i \right) \right\}$$

Experimental Setup

Fitness Measure: Points

- $m = 100$ for satisfied Target Objectives (ω)
- $m' = 80$ and $m'' = 10$ for Satisfied Partial Objectives (ω' and ω'')

Parameters for Simulated Annealing

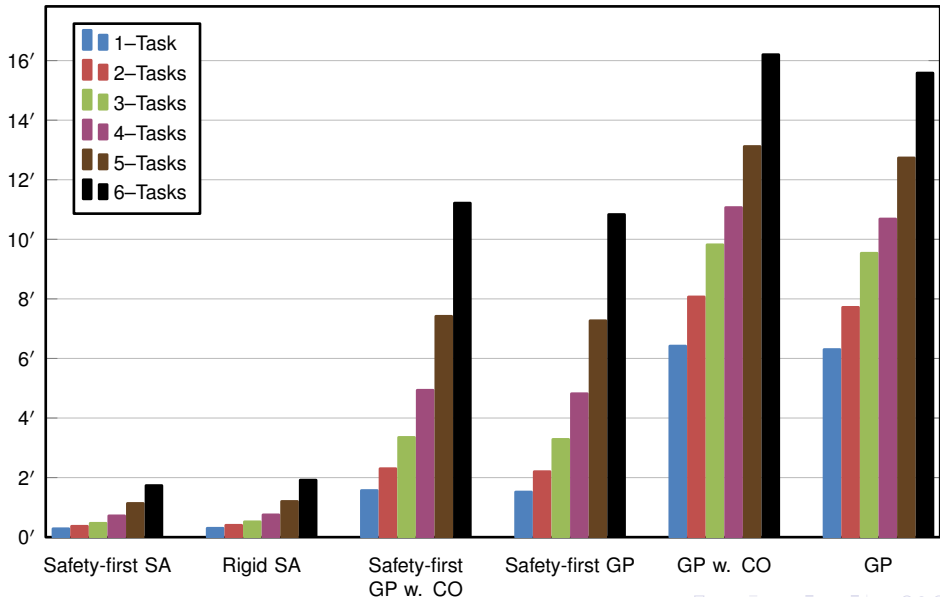
- Initial temperature of 20,000; Decreasing by 0.8 at each iteration
- \rightsquigarrow Search ends after 25,000 iterations

Parameters for Genetic Programming

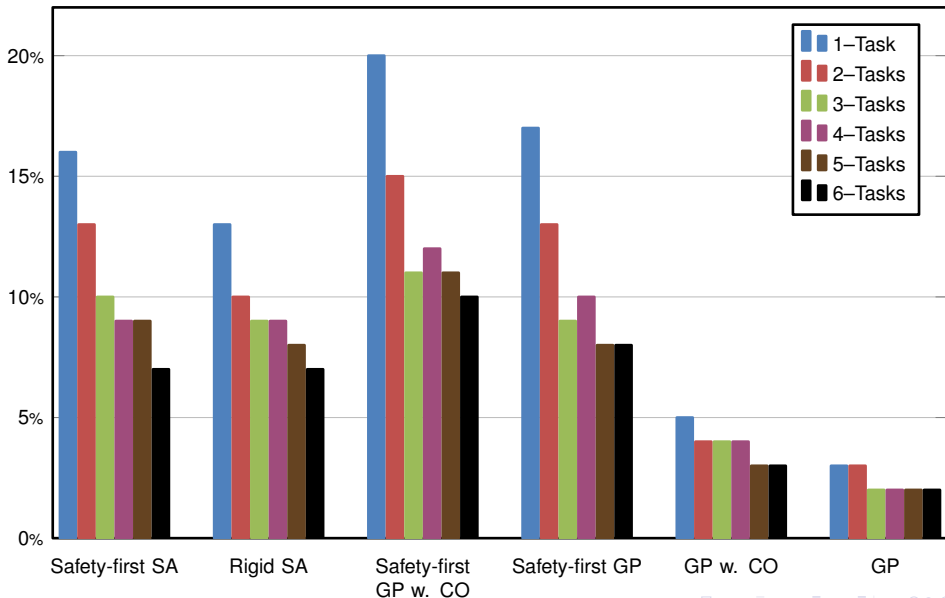
- Population of 150 candidates; Keep 5 at each iteration
- Abort after 2,000 iterations

- Using NuSMV Model-checker
- Running on Intel core i7 3.40 GHz CPU with 16GB RAM

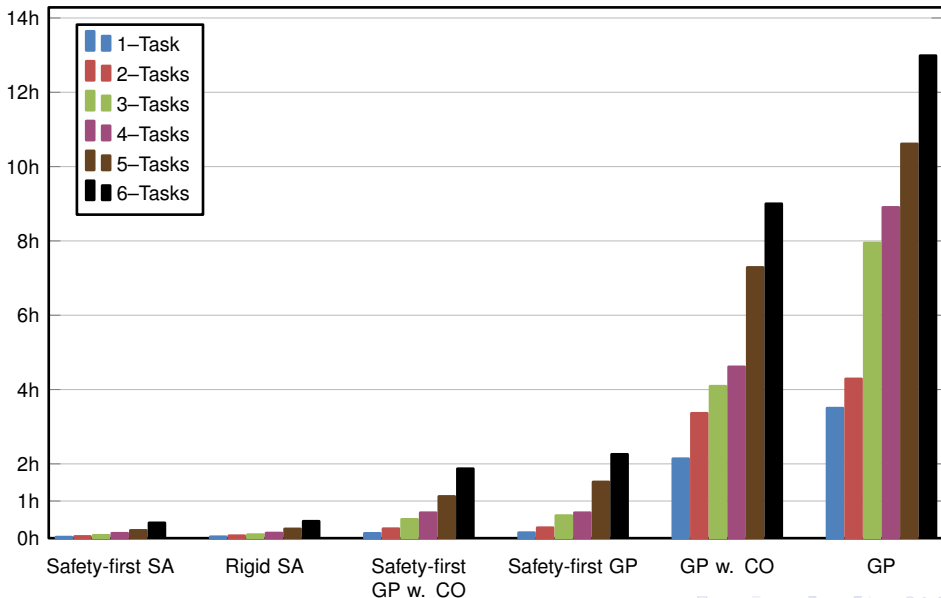
Experimental Results: Individual Execution Time



Experimental Results: Success Rate



Experimental Results: Overall Expected Runtime



Summary

- Solving a Class of DCS Problems where Deterministic Strategies are sought with General Search Techniques
- Experimental Evaluation using a Scalable DCS Problem
- Relative Performances of the Algorithms match our previous results for Program Synthesis
- Runtime Performances of Classical Symbolic Techniques are several orders of magnitude better [[Marchand et al., 2000](#), [Berthier and Marchand, 2014](#)]

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Trails for Improvements & Further Investigations

- Parameter tuning: Good cooling schedule? Population size?
- Integration in a Model-checker

Thanks!

- N. Berthier and H. Marchand. Discrete Controller Synthesis for Infinite State Systems with ReaX. In [12th Int. Workshop on Discrete Event Systems](#), WODES'14, pages 46–53. IFAC, May 2014. ISBN 978-3-902823-61-8.
- J. A. Clark and J. L. Jacob. Protocols are programs too: the meta-heuristic search for security protocols. [Information and Software Technology](#), 43:891–904, 2001.
- D. Henderson, S. H. Jacobson, and A. W. Johnson. The theory and practice of simulated annealing. In [Handbook of metaheuristics](#), pages 287–319. Springer, Berlin, Heidelberg, 2003.
- I. Husien and S. Schewe. Program generation using simulated annealing and model checking. In [International Conference on Software Engineering and Formal Methods](#), pages 155–171, Berlin, Heidelberg, 2016. Springer.

- C. G. Johnson. Genetic programming with fitness based on model checking. In [EuroGP](#), volume 4445 of [LNCS](#), pages 114–124, Berlin, Heidelberg, 2007. Springer.
- G. Katz and D. Peled. Model checking-based genetic programming with an application to mutual exclusion. In [TACAS](#), volume 4963 of [LNCS](#), pages 141–156, Berlin, Heidelberg, 2008. Springer.
- G. Katz and D. Peled. Model checking driven heuristic search for correct programs. In [MoChArt](#), volume 5348 of [LNCS](#), pages 122–131, Berlin, Heidelberg, 2009. Springer.
- G. Katz and D. Peled. Synthesizing solutions to the leader election problem using model checking and genetic programming. In [Proceedings of the 5th International Haifa Verification Conference on Hardware and Software: Verification and Testing](#), HVC'09, pages 117–132, Berlin, Heidelberg, 2011. Springer. ISBN 978-3-642-19236-4.

- H. Marchand, P. Bournai, M. Le Borgne, and P. Le Guernic. Synthesis of discrete-event controllers based on the signal environment. [Discrete Event Dynamic System: Theory and Applications](#), 10(4): 325–346, Oct. 2000.